Comparison and Analysis of Equalization Techniques for the Time-Varying Underwater Acoustic Channel

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• Introduction:
  – Underwater Communication
  – Decision Feedback Equalization
    • Channel Estimate Based
    • Direct Adaptation
• Analysis of Equalization Behavior
• Simulation Results
• Summary and Conclusion
• Future Directions
Underwater Communication
Time Varying Impulse Response

![Image of impulse response graph]

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Channel Model

\[ d[n] \sum_{k=-N_\alpha}^{N_c-1} g^*[n, k]d[n - k] + v[n] \rightarrow u[n] \]
Channel Model (cont.)

Vector-form: \[ g^H[n]d[n] + v[n] = u[n] \]

\[
\begin{bmatrix}
g^*[n,-N_c+1] & g^*[n,N_c+2] & \ldots & g^*[n,0] & \ldots & g^*[n,N_a]
g^*[n,-N_c+1] & \ldots & \ldots & \ldots & \ldots & \ldots
0 & 0 & \ldots & 0 & \ldots & 0
g^*[n-L_a+1,N_c-1] & \ldots & g^*[n-L_a,1,0] & \ldots & g^*[n+L_a-1,N_a] & 0 & \ldots & 0 & \ldots & 0
0 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0
\end{bmatrix}
\begin{bmatrix}
d[n-N_c+1] \\
\ldots \\
d[n] \\
\ldots \\
d[n+N_a]
\end{bmatrix}
\rightarrow
\begin{bmatrix}
\vdots \\
v[n] \\
\vdots
\end{bmatrix}
\rightarrow
\begin{bmatrix}
u[n-N_c+1] \\
\ldots \\
v[n] \\
\ldots \\
u[n+N_a]
\end{bmatrix}

Equalization

TX Data bit (linear) estimator:

\[ \hat{d}[n] = h^H[n]z[n] \]

Vector of RX data and TX data estimates

**LMMSE Optimization:**

\[ \hat{h}_{opt} = \arg \min_{h'} E\{|h'^H z - d|^2\} \]

**Solution:**

\[ \hat{h}_{opt}[n] = R_z^{-1}[n]r_{zd}[n] \]

\[ R_z[n] = E\{zz^H\} \]

\[ r_{zd} = E\{zd^*\} \]

**Recursive Processing (lag 1):**
Decision Feedback Equalizer (DFE)

- Two Parts:
  - (Linear) feed-forward filter (of RX data)
  - (Linear) feedback filter (of data estimates)
- Estimate using RX data and TX data estimates
  \[ z[n] = [u[n-Lc+1] \ldots u[n] \ldots u[n+La], \hat{d}[n-1] \ldots \hat{d}[n-L_{fb}]]^T \]
- Split Channel convolution Matrix:
  \[ G \leftrightarrow \begin{array}{c} G_0 \\ G_{fb} \end{array} \]
  - Received data becomes: \[ u = G_0d_0 + G_{fb}d_{fb} + v \]
- Minimum Achievable Error:
  \[ \sigma_{0,df\epsilon}^2 = 1 - g_0^H[G_0G_0^H + R_v]^{-1}g_0 \]
DFE: Direct Adaptation

\[
h = \begin{bmatrix} h_{ff}[n] \\ h_{fb}[n] \end{bmatrix} = E\{zz^H\}^{-1}E\{zd^*\}
\]
DFE: Channel Estimate

\[
\begin{align*}
\mathbf{h}_f &= \left[ \mathbf{G}_0 \mathbf{G}_0^H + \mathbf{R}_v \right]^{-1} \mathbf{G}_S \\
\mathbf{h}_{fb} &= -\mathbf{G}_{fb}^H \mathbf{h}_f
\end{align*}
\]
Assumptions

• Unit variance, white transmit data
  \[ E\{d[n]d^H[n]\} = I \]

• TX data and obs. noise are uncorrelated
  \[ E\{v[n]d^H[m]\} = 0 \]
  
  – Obs. Noise variance:
  \[ R_v = E\{v[n]v^H[n]\} \]

• Perfect data estimation (for feedback)
  \[ \hat{d} = d \]

• Equalizer Length = Estimated Channel Length
  \[ N_a + N_c = L_a + L_c \]

• MMSE Equalizer Coefficients have form:
  \[ h_{ff} = [G_0G_0^H + R_v]^{-1}G_S \]
  \[ h_{fb} = -G_{fb}^H h_{ff} \]
Comparison between DA and CEB

• In the past, CEB methods empirically shown to have lower mean squared error at high SNR

• Reasons for difference varied:
  – Condition number of correlation matrix
  – Number of samples required to get good est.

• Analysis to follow: low and high SNR regimes
Comparison of DA and CEB on Rayleigh Fading Channel

BER Equalizer Comparison on a 5-tap Rayleigh Fading Channel

SDE Equalizer Comparison on a 5-tap Rayleigh Fading Channel
Why the difference?

- Correlation time
  - DA equalizer taps have lower correlation time at high SNR
  - At low SNR, two methods equivalent

- But how do we show this?
  - Combination of analysis and simulation
AR channel model

- Simple channel model to analyze
- Similar to encountered situations

\[ g[n + 1] = \alpha g[n] + w[n] \]

\[ R_{gg}[k] = E\{g[n]g^*[n + k]\} = \begin{cases} \sigma_w^2 \left( \frac{(\alpha^*)^k}{1 - |\alpha|^2} \right) & k \geq 0 \\ \sigma_w^2 \left( \frac{\alpha^{-k}}{1 - |\alpha|^2} \right) & k < 0 \end{cases} \]
\[ h_{ff}[n + 1] = (G_0[n + 1]G_0^H[n + 1] + R_v)^{-1}(g_0[n + 1]) \]

\[ \approx R_v^{-1} (\alpha g_0[n] + w[n]) \]

\[ \approx \alpha h_{ff}[n] + R_v^{-1}w[n] \]

\[ R_v + G[n]G^H[n] \approx R_v \]
\[ h_{ff}[n + 1] = (G_0[n + 1]G_0^H[n + 1] + R_v)^{-1}(g_0[n + 1]) \]

\[ Q[n] = G_0[n]G_0^H[n] + R_v \approx G_0[n]G_0^H[n] \]

\[ (G_0[n]G_0^H[n])h_{ff}[n] = g_0[n] \]
\[(G_0[n]G_0^H[n])h_{ff}[n] = g_0[n]\]

\[G_0 = \begin{bmatrix}
g_0^*[n - L + 1] & 0 & 0 & \cdots & 0 \\
g_1^*[n - L + 2] & g_0^*[n - L + 2] & 0 & \cdots & 0 \\
g_2^*[n - L + 3] & g_1^*[n - L + 3] & g_0^*[n - L + 3] & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
g_L^*[n] & g_{L-1}^*[n] & g_{L-2}^*[n] & \cdots & g_0^*[n]
\end{bmatrix}\]

\[G_0G_0^H = \begin{bmatrix}
|g_0|^2 & \cdots \\
g_0g_1^* & \cdots \\
g_0g_2^* & \cdots \\
\vdots & \ddots \\
g_0g_L & \cdots
\end{bmatrix} \quad h[n] = \begin{bmatrix}
1/g_0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}\]
Correlation over SNR

AR(1) model

Gaussian model
Multi-tap AR(1) model
Conclusions

• As SNR increases, correlation time of equalizer taps is reduced
  – CEB is tracking value correlated over longer time
  – DA should do worse

• Assumed noise statistics were stationary
  – Not always case in underwater

• Underwater communication is power limited
  – Operate in low SNR regime (<35dB)
Future Work

• Include channel state information into DA
  – Sparsity

• Reduce number of snapshots for channel model
  – Physical constraints?
  – Compressed sensing?
Questions?